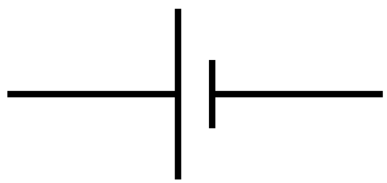
DC circuit basics *(review)*

A DC power supply is designed to produce an electric field in one direction through the wires connected to the supply. That field motivates charge carriers to move in one direction and one direction only in the circuit.

The symbol for a DC power supply is:



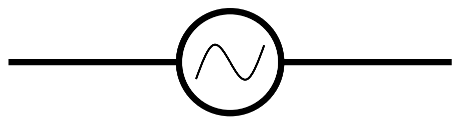
How does this occur?

- *DC power is* produced by setting up a fixed potential difference between two terminals using a battery or other such source.
- *The idea of current* in a DC setting makes perfect sense. As charge carriers move in one direction and one direction only, a count of the number of charge carriers that pass by a point per unit time gives us the current in the branch.

AC circuit basics

An AC power supply is designed to produce an electric field that alternates back and forth in direction through the wires connected to the supply. That field motivates charge carriers to jiggle back and forth in the circuit.

The symbol for an AC power supply is:



How is this produced?

- By rotating a coil in a magnetic field! (Remember…?)

The concept of current in a AC setting is less obvious.

- *As charge carriers move* back and forth, a count of the number of charge carriers that pass by a point per unit time yields zero net ("average") current in the branch. As we do want to assign current-like information to AC circuits, how we can do this is something we have to discuss in more detail.

Making sense of AC current

Remembering that a battery's voltage in a DC circuit really tells us the potential difference between the battery's terminals (in other words, it's really a ΔV term), the function that identifies the "voltage" across an AC source must tell us what the potential difference is between the AC source's terminals. What makes this a bit exciting is the fact that this voltage *changes with time*.

The time dependent function that identifies the voltage across an AC source's terminals is:

$$V(t) = V_o \sin(\omega\ t)$$

where V_o is the amplitude of the voltage (i.e., the largest voltage the source can provide), ω is the angular frequency of the source and identifies how fast the function is changing, and the product ωt is an angular measure in radians (sine functions act on angles).

Making sense of AC current

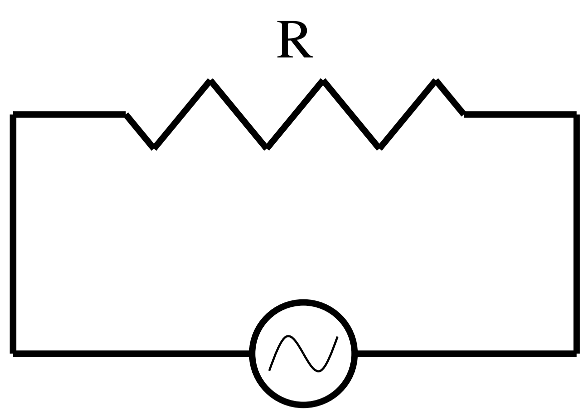
What is the relationship between ω and ν?

$$\omega = 2\pi\nu$$

So we can rewrite the voltage function as:

$$V(t) = V_o \sin(2\pi\nu\, t)$$

This means a simple AC circuit can be represented like this:



$$V(t) = V_o \sin(2\pi\nu\, t)$$

Quick note for household circuits

In the US, most household power has a frequency of 60 Hz, so the standard voltage function becomes:

$$V(t) = V_o \sin\left(2\pi(60)\ t\right)$$
$$= V_o \sin\left(377\ t\right).$$

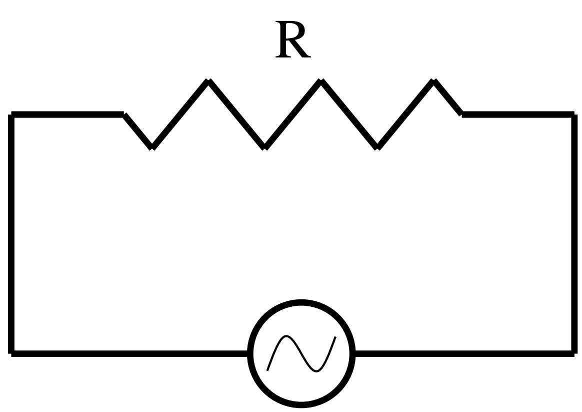
– *This is only good* if we KNOW the frequency is 60 Hz.

We still don't know V_o (no, it's NOT 120 V), but we'll deal with that in a minute…

Making sense of AC current

So consider that standard AC circuit we just saw. Assume the resistor is a light bulb, and the AC source is providing power to light the bulb.

The question: How large would a DC battery have to be, and how much DC current would that battery have to supply to the circuit so that the power being provided to the circuit was the same as the power being provided by the AC source?



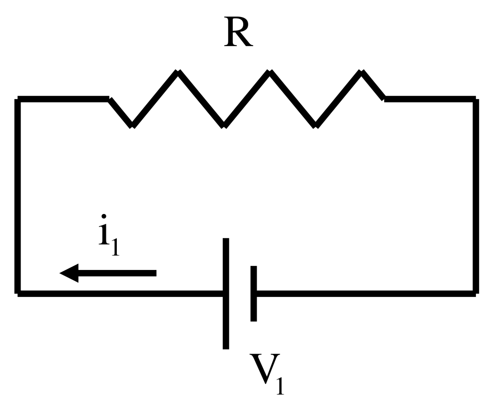
$$V(t) = V_o \sin(2\pi\nu\, t)$$

AC vs DC equivalent current

The key is to determine the DC-equivalent voltage and current (the DC voltage and current that mimics the AC voltage and current) and to do that we need to look at the power relationship.

In a DC circuit, the power being provided by the battery is equal to:

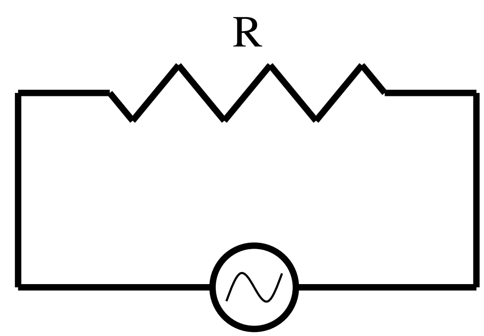
$$P = (i_1)^2 R,$$



Following suit for our AC circuit, the AC source's power must equal:

$$P = (i_{DC\ effective})^2 R,$$

where the $i_{DC\ effective}$ quantity is related to the AC current but is additionally equal to the amount of DC current needed to parallel the AC case.

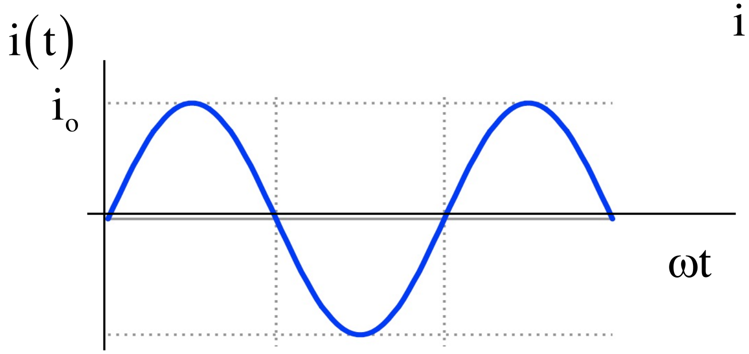


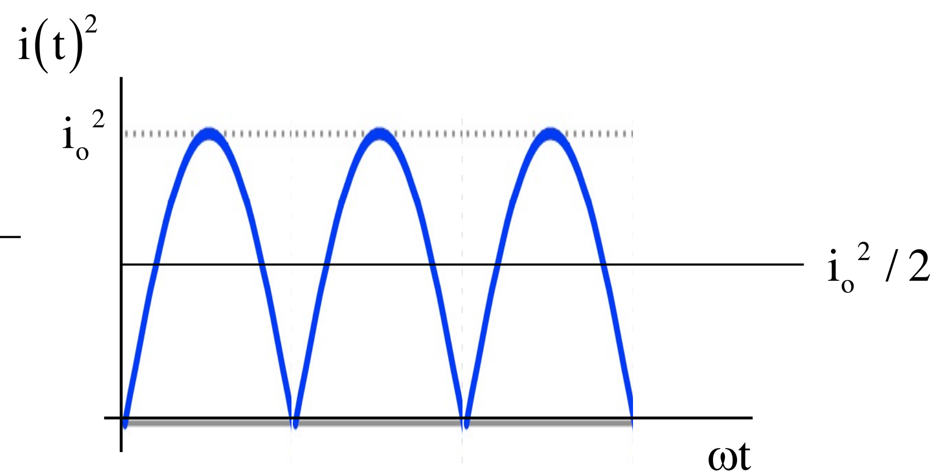
$I_{dc\ equivalent}$

Reiterating, $i_{DC\text{-}equivalent}$ is the amount of current you would need in a DC circuit to duplicate the power input being provided by the AC source. But how to calculate that quantity? The average of the current in the AC circuit will be zero.

What we really want is not the average current, it's the average of the current *squared*. The *current squared* is what determines the power.

The square of a sine wave function is always positive, and although it doesn't look so in my sketch the walls of a sine function squared are fairly vertical. As such, we can approximate the average of the AC current-squared as the amplitude of the current-squared divided by 2.





$I_{dc\ equivalent}$ aka I_{RMS}

From this, we can write:

$$(i_{DC-equiv})^2 = \frac{i_o^2}{2}$$

$$\Rightarrow \sqrt{(i_{DC-equiv})^2} = \sqrt{i_o^2/2}$$

$$\Rightarrow i_{DC-equiv} = \frac{i_o}{\sqrt{2}}$$

$$\Rightarrow i_{DC-equiv} = 0.707 i_o$$

As the process used to get this relationship was to take the square **R**oot of the **M**ean value of the amplitude **S**quared, it's called the RMS value of the AC current. In other words:

$$i_{RMS} = .707\ i_o$$

RMS Voltage

Likewise, there is a RMS value for voltage, also. It is written as:

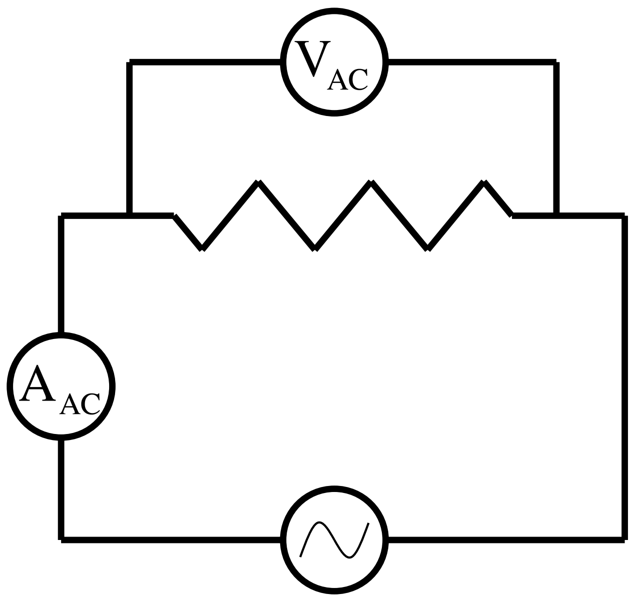
$$V_{RMS} = .707\ V_o$$

It is important to understand what RMS values give you. In the case of voltage generated by an AC power supply, it gives you the single, DC voltage that would provide to the circuit *the same amount of power* the AC power supply does. In other words, if you wanted to take the AC source out of the circuit and replace it with a comparable DC source, that's how big the DC source would have to be to accommodate the job.

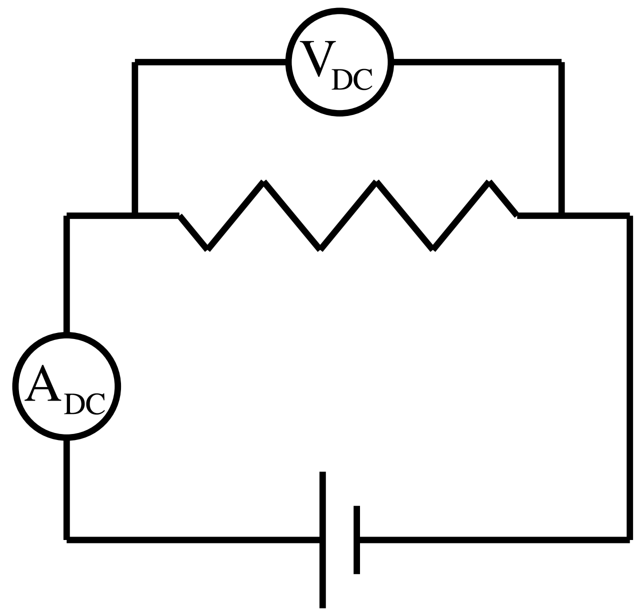
Additionally, as a DC meter in an AC circuit would just quiver (the alternating electric field would make charge go one way, then the other way, etc., and the DC meter wouldn't be able to keep up with the changes), AC meters are designed differently than DC meters. In any case, **AC meters read RMS values**.

AC vs DC circuits

The meters in the two circuits below will read the same (assuming the resistance is the same in both circuits). (See if you can't see why.)

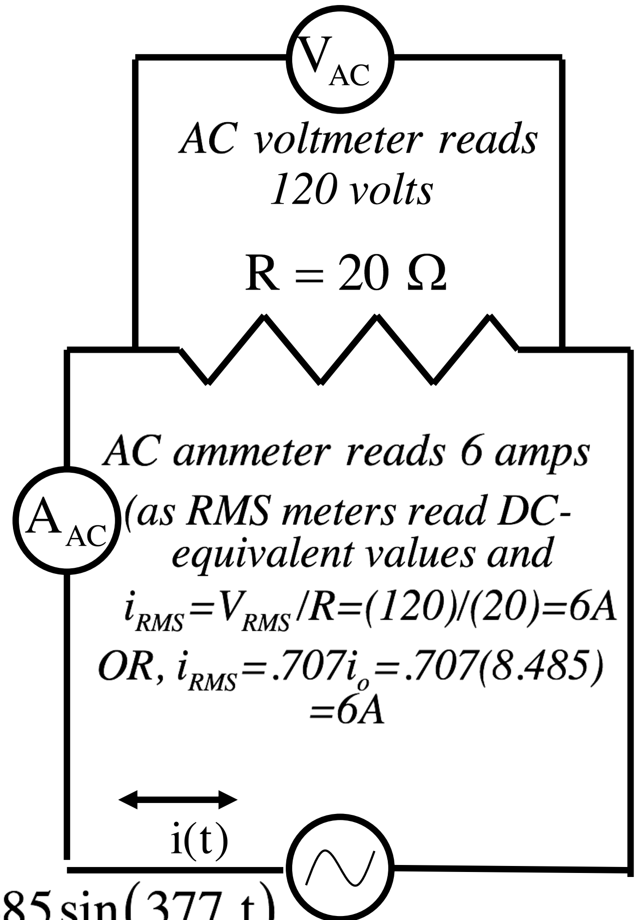


$$V(t) = V_o \sin(2\pi\nu\, t)$$



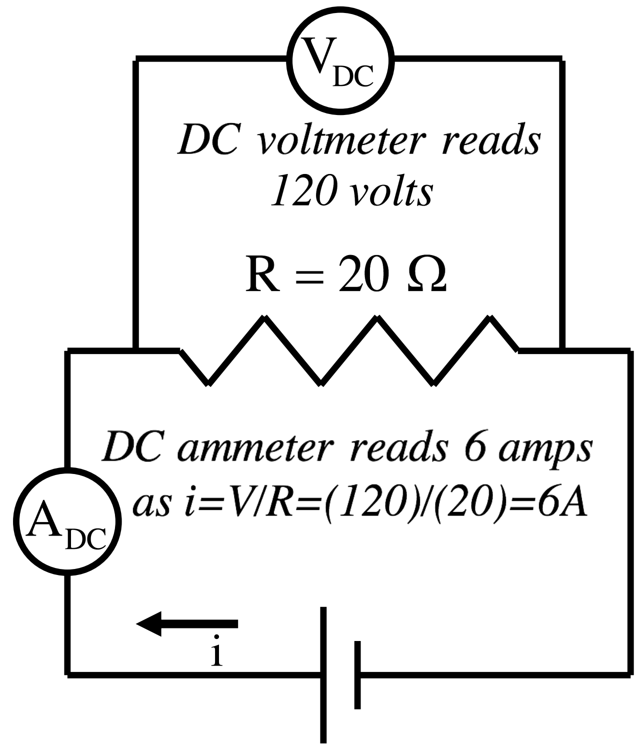
$$V_{RMS} = .707V_o$$

Another example



$$i(t) = 8.485 \sin(377\ t)$$

$$V(t) = 169.7 \sin((2\pi(60\ Hz))\ t)$$



$$V_{DC} = 120 \text{ volts}$$

$$as\ V_{RMS} = .707(169.7) = 120\ volts$$

In short, you can start with the DC-equivalent values and determine their AC counterparts, or go the other way.

13.31) Charge carriers in a DC circuit move in one direction only. What do charge carriers do in an AC circuit?

13.32) The idea of *current through* and *voltage across* a resistor in a DC circuit is fairly straightforward. Current measures the amount of charge that passes through the resistor *per unit time*, and voltage measures the unchanging voltage difference between the two sides of the resistor. The idea of a resistor's current and voltage in an AC circuit is a little more complex, given the fact that the charge carriers in AC circuits don't really go anywhere. So how do we deal with the idea of current and voltage in an AC circuit? That is, when someone says that your home wall socket is providing 110 volts AC, and that a light bulb plugged into that socket draws .2 amps of current, what are those numbers really telling you?

13.33) An AC voltage source is found to produce a 12 volt peak to peak signal at 2500 hertz.

- **a.)** Characterize this voltage as a *sine function.*
- **b.)** Graph the *voltage versus time* function.
- **c.)** Determine the *RMS voltage* of the source and put that value on your graph from *part b.*
- **d.)** It is found that an ammeter in the circuit reads 1.2 amps. What is the *maximum current* drawn from the source?